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Separation of contact in a sliding system with frictionally excited thermoelastic instability

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Abstract

The first phase of the thermoelastic instability, which is characterized by a full contact regime, can be modelled using Fourier decomposition and application of an analytical description. However, in case of further increase of instability, a separation of the contact occurs which is more difficult to cover by mathematical means. The contribution deals with numerical simulations of the separation. The problems are topical in connection with the disk brakes design, should we give an example.

Keywords: sliding contact, thermoelastic instability, hot spots

1. Introduction

The frictionally excited heating of brake components and multidisk clutches causes thermoelastic deformations of the contact bodies. As a rule, this effect leads to redistribution of the initial contact pressure. Are the velocities of the contact slip rather high, the equipments can display unwanted behaviour. In particular, let us mention the origin of hot spots, vibrations, enhanced wear, up to damaging the material. A fatal loss of the equipments functionality can happen.

Barber has called the thermoelastic instability (subsequently denoted as TEI) a cause of such effects in his work [1]. At some later time, Dow and Burton [4] and Burton et.al. [2] designed a mathematical model that enabled understanding the core of the effect in more detail and that made it possible to assess the influence of both material parameters and the sliding velocity on a rise and development of the instability. They considered however only half-planes (or half-spaces) to be contact bodies thus losing a chance to evaluate the influence of geometrical dimensions such as thicknesses of disks and friction pads. An analytical solution of the problem in question, described by partial differential equations, either gets rightly complicated or is impossible to be expressed. On that account, Lee and Barber [6] produced an analytical periodical solution that involves behaviour of a disk (infinite layer) clamped between half-spaces as late as after 20 years. The demand on further development of analytical approach to make the models of brakes and clutches more precise is therefore plain enough (see e.g. [3], [10]).

The suggested range of analytical problems has some essential limitations. The first one lies in the fact that the intermittent contact (the friction pads occupy only a part of the disk circumference) cannot be appreciated exactly, thus making necessary to choose a convenient averaging method (see [10]). Besides it, it is impossible to consider the material parameters

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and the friction coefficient as time-dependent functions. Finally, a major limitation is the assumption of the full contact regime between the pads and the disk imposed on the analytical solutions. Since the time-dependent rise in perturbation during the TEI is exponential, then – supposing that the parameters of the model (sliding velocity above all) remain constant – it is only a matter of time before the contact separation occurs. Hence, the initial contact between the friction pads and the disk reduces. Zagrodski et al. [11] were the first to devise a numerical approach that would cover both the above-stated limitations and the transients connected with the TEI. They made use of the Petrov-Galerkin method as a modification of the finite elements implemented in the system ABAQUS. Their work brought substantial benefits and it suggests the complexity of the actual state of things associated with fully non-linear behaviour. Another work worthy of note is [12].

It is mainly the transport engineering that urges the effort to investigate and explain the TEI effects by numerical simulations. Wide-ranging measurements were performed when developing the brakes for the high-speed train TGV (cf. [8]); even in the automobile industry the growth of hot spots and manifestation of the TEI are investigated (see e.g. [5] or an experimental methodology development project at the research centre NTC of the University of West Bohemia [7]). A number of experimental findings associated with behaviour of disk brakes appear not to be theoretically or computationally clarified yet.

The goal of this contribution is drawing the attention to some issues about numerical simulations of the TEI. Its author makes here use of a program code of his own authorship, written and debugged in the programming language FORTRAN. An attention is also given both to the influence of the initial perturbation and to the temperature dependence of the friction coefficient on the instability development.

2. Mathematical model

Sliding contact between the disk and the friction pads is, in a simplified fashion, modelled as a planar problem by their unfolding. It is assumed for the third direction (actually the radial direction of the disk brake or the clutch) that the temperature and deformation are not functions of the corresponding coordinate. It is sufficient for the purposes of simulation to consider the contact length as corresponding to the wave length L of the periodic solution. A coordinate system (x,y), firmly connected with the disk, has been introduced in so far that the sliding friction pads with the sliding velocity V (see Fig. 1) have been incorporated into the mathematical model. Some numerical circumstances (see [11]) were the reason for such a choice. For the simplicity we suppose for the rest of this paper that the velocity V is time-independent.

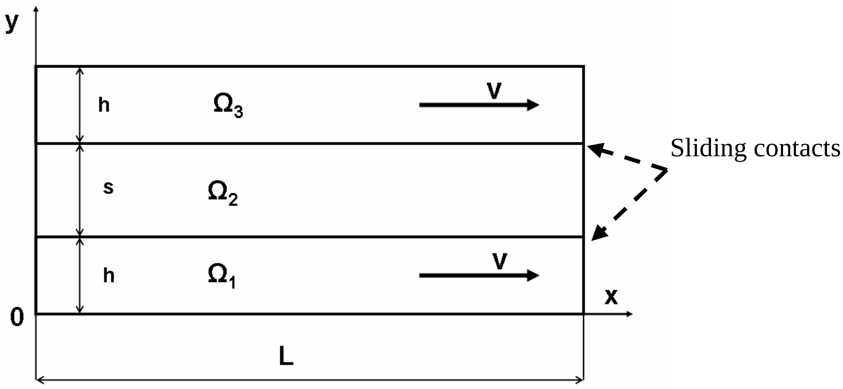

Fig. 1. Geometry of the model of a pair of sliding contacts.

2.1. Heat transfer

The heat transfer balance in the central layer is given by the equation

$$\frac{\partial T_2}{\partial t} = k_2\,\Delta T_2 \;,$$

where $T_2(x,y,t)$ is a temperature field in the layer 2 (region Ω_2), k_2 is the temperature diffusivity of this layer, t is the time, and Δ denotes the Laplace operator. For moving layers 1 and 3, heat conduction equations involve yet a convective term. Using analogous notation and considering the sliding velocity to be V, we have

$$\frac{\partial T_i}{\partial t} + V\frac{\partial T_i}{\partial x} = k_i\,\Delta T_i \;,\quad \mathrm{i} = 1, 3.$$

The heat flow q_{ij} generated by a contact between the layers i and j will be

$$q_{ij} = f\,V\,p_{ij}\,,$$

where p_{ij} denotes a corresponding contact pressure and f is friction coefficient whereas

$$q_{12} = -K_1\frac{\partial T_1}{\partial y} + K_2\frac{\partial T_2}{\partial y} \quad \text{for } y_{12} = h \;\text{ and }\; q_{23} = -K_2\frac{\partial T_2}{\partial y} + K_3\frac{\partial T_3}{\partial y} \quad \text{for } y_{23} = h+s\,.$$

It is further necessary to ensure the temperatures of contact surfaces to be equal. If we denote the distance between opposite points on the surfaces of the layers i and j by $w_{ij}(\mathrm{x,t})$, then

$$T_i = T_j\,,\quad \text{if } w_{ij} = 0\,.$$

The conditions of periodicity of the solution $T_i(0,y,t) = T_i(L,y,t)$, will then be satisfied on the vertical boundaries while the adiabatic condition of zero heat flow is assumed for the horizontal boundaries.

2.2. Elastic deformations

We assume that the contribution of incidental vibrations of the layers inertia mass to the elastic deformations is negligible. The magnitudes of elastic waves of the materials in question are also large as compared with the assumed sliding velocity V. Accordingly we obtain elastic deformations as a solution of quasi-static contact plane strain problem which matches best the situation. The loading applied is partly a pressure p_m acting on the outside horizontal boundaries of the friction layers and partly temperature fields T_1, T_2 and T_3 at the time point t. The size of the gap $w_{ij}(x,t)$ and the pressure $p_{ij}(x,t)$ between the layers i and j satisfy the conditions

$$p_{ij} \ge 0\,,\; w_{ij} = 0 \text{ for } p_{ij} > 0\,,\; w_{ij} \ge 0 \text{ for } p_{ij} = 0\,,\; w_{ij} = u_{y,j}(x,y_{ij},\mathrm{t}) - u_{y,i}(x,y_{ij},\mathrm{t})\,,\; \mathrm{ij} = 12 \text{ or } 23,$$

where $y_{12} = h$, $y_{23} = h+s$ and where $u_{y,j}(x,y,t)$ denotes a displacement of the point (x,y) of the layer j in the direction of the y axis. The displacements satisfy the periodicity conditions

$$u_{x,i}(0,y,t) = u_{x,i}(L,y,t) + const\,,\; u_{y,i}(0,y,t) = u_{y,i}(L,y,t)\,,\quad \mathrm{i} = 1,2,3\,,$$

on vertical boundaries whereas the constant *const* is implied by the thermal deformations and is not known before the problem solution gets started. It yields

$$u_{y,i}(x,0,t) = 0\,,\; u_{y,i}(x,2h+s,t) = const\,,\quad \mathrm{i} = 1,2,3\,,$$

for the outside horizontal boundary where the other constant *const* is again a result of the overall compression.

3. Thermoelastic instability

We assume in this paper that both the outside pressure p_o acting on the friction layers and the sliding velocity V are time-independent. Let us further consider an initial perturbation with an amplitude p_a that can be caused by an initial disk corrugation or from some other reason. As a result, this perturbation violates the constant contact pressure and the temperature field in the direction of the x axis. Sufficiently high velocity V brings on thermoelastic instability that finds expression in growth in the perturbations of the contact pressure amplitudes and of the temperature. Conversely, the perturbations are damped down at low velocities V. The contact pressure can be expressed in the form

$$p(x,t) = p_o + p_a \exp(bt)\, \Psi\left[\frac{2\pi}{L}(x + ct)\right]$$

for the initial phase of the full contact regime where b is a growth rate that characterizes the instability, and c is so called migration speed. If b<0, the system is stable, for b>0 instable. The expression for temperature field is similar and it includes the term exp(bt) too. The function Ψ describes elementary perturbation mode (at least in the initial phase of the full contact regime), general perturbation can be decomposed into a Fourier series with these elementary terms. Each term of course has its own magnitude of parameters b and c). The function Ψ can be approximated sufficiently closely by the goniometric function sinus in extreme, fictive case of negligible shearing force.

If a numerical approach is the case, we know the values of $p(x,t)$ and p_o. If $p_{max}(t)$ denotes the maximum pressure on the contact surface, the parameter b can be found using the formula

$$b = \ln[\, p_{max}(t+\Delta t)/p_{max}(t)\,]/\Delta t\,.$$

4. Numerical simulation

The Petrov-Galerkin method for solving the heat problems was implemented in the way described in [9]. Elastic displacements and deformations have been calculated in a usual manner using the finite element method. The above mentioned contact conditions and boundary conditions of solution periodicity have been implemented in terms of the penalty method. A solver of the library CXML has been used for solving the system of linear algebraic equations with sparse matrix.

Let us consider an example with very large parameter b as an extreme case. Let the dimensions of the model be therefore chosen as h = 4 mm, s = 6 mm, L = 40 mm. In order to keep up the elementary mode close to the function sinus, we have suppressed tangential contact forces which can be analytically formulated by setting the Poisson number to 0.5 (actually, it was chosen 0.495 for the computation). Furthermore, p_o = 2 MPa was considered in the calculation, the material parameters are presented in the Tab. 1, friction coefficient f = 0.4, and the sliding velocity V = 5.5 m/s.

	Modulus of elasticity	Thermal expansion coefficient	Thermal conductivity	Spedific heat	Density
	E [MPa]	α [1/^{o}C]	K [W/m^{o}C]	c [J/kgK]	ρ [kg/m^3]
Friction layers	1000	1.0E-05	5.0	35.0	4000
Disk	125000	1.2E-05	54.0	53.3	7800

Tab. 1. Material parameters of the model.

The initial perturbation was applied in the form of contact pressure perturbation, it acted for the duration of 0.025 s, it had a shape of the sinus function with amplitude 0.1 MPa and it was stationary with respect to the disk. Analytical solution found in the way following the methodology [10] gives the value of the parameter $b \approx 21\ s^{-1}$ in the investigated case while the numerical solution using the finite element method yields $b \approx 24\ s^{-1}$. Consequently, the error does not exceed 15 %. Such a result can be judged satisfactory since the example is extreme one being featured by large temperature gradients, particularly in the contact surface layer of the friction pad. In addition, the discretization through rectangular finite elements was efficient. Their size was 1 mm resp. 0.083 mm (pad) or 0.166 mm (disk) in the direction of the *x*-axis resp. in the *y*-direction at the contact. The development of both contact pressures and temperatures are shown in the Fig. 2. The value of the critical velocity V_{cr} is 1.2 m/s for the given geometry and the material parameters. This sliding velocity implies $b = 0$.

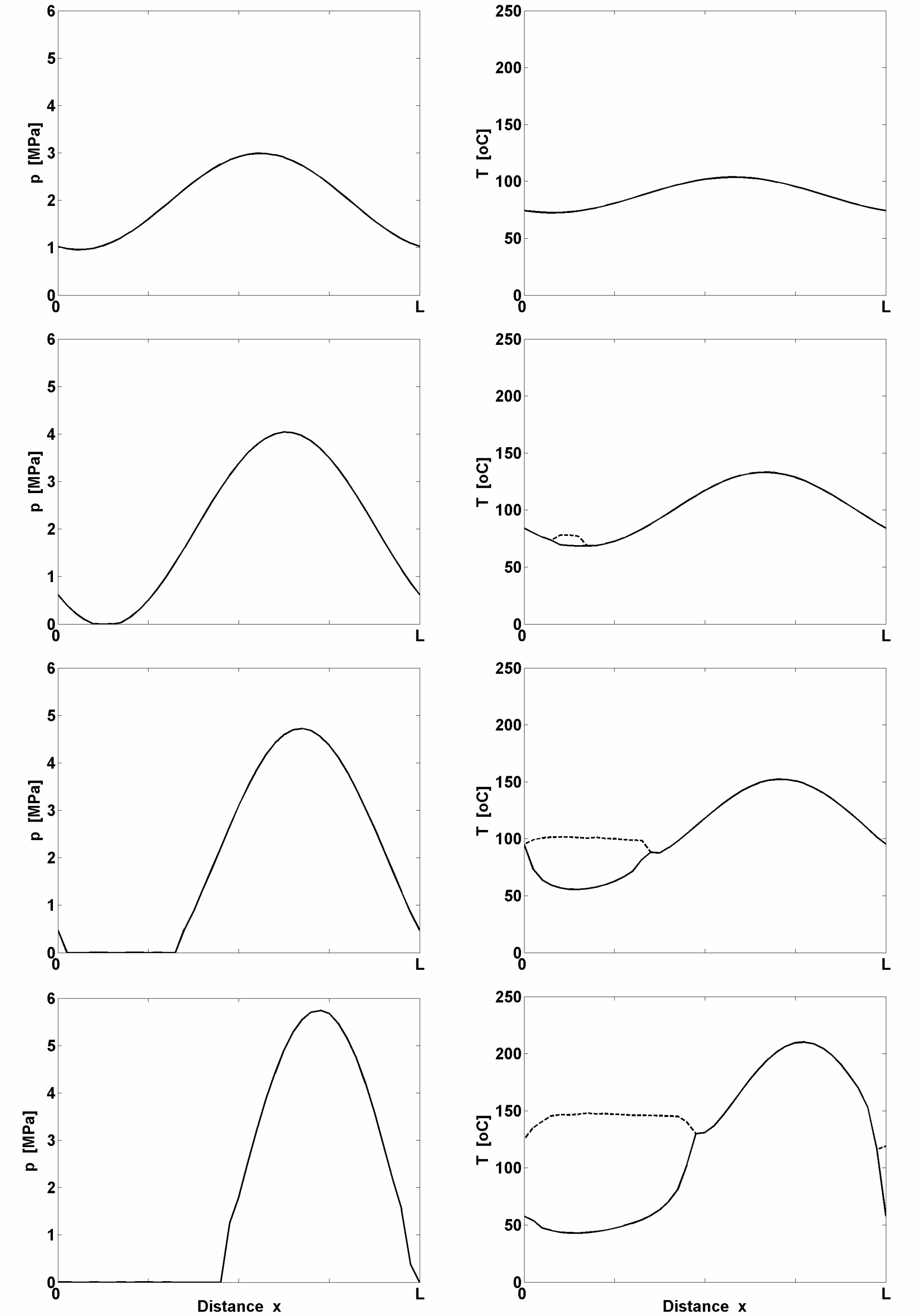

Fig. 2. The developments of contact pressure and surface temperatures for the time points 0.10 s, 0.13 s., 0.15 s and 0.20 s in the course of thermoelastic instability in the described task (*full line* plots the disk surface temperature while *dashed* is that of friction pad).

5. Discussion on current problems

The numerical approach, as against the analytic one, enables to model the course of the thermoelastic instability more realistically with respect to the conditions that can be found in the technical systems. Particularly the question is second phase of the instability development when the contact gets separated. The possibility to consider material parameters to be temperature-dependent is also essential. If, for example, the friction coefficient decreases to the value of 0.2 from 0.4 at the temperature of 400°C, the separation of the contact occurs at the time-point 0.172 s vs. 0.13 s. This means that its arrival is approximately 30 % slow. It is also apparent that we can solve even some considerably extreme situations using Petrov-Galerkin method.

The character of the initial perturbation implementation affects the development of the instability too. With the actual devices, the initial non-planeness of the disk is usually crucial. This can be taken into account in the numerical simulations by introducing an additional fictive temperature field that would produce a corresponding additional deformation. A similar effect can be invoked if we bring in the perturbation into the model with the help of the pressure as we have already explained in the above, but we will not cut it off in the time.

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